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Asymmetric superconductivity in metallic systems

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Abstract
Different types of superfluid ground states have been investigated in systems of two species of fermions with Fermi surfaces that do not match. This study is relevant for cold atomic systems, condensed matter physics, and quark matter. In this paper we consider this problem when the fermionic quasi-particles can transmute into one another and only their total number is conserved. We use a Bardeen–Cooper–Schrieffer (BCS) approximation to study superconductivity in two-band metallic systems with inter-and intra-band interactions. Tuning the hybridization between the bands varies the mismatch of the Fermi surfaces and produces different instabilities. For inter-band attractive interactions, we find a first-order normal–superconductor transition and a homogeneous metastable phase with gapless excitations. In the case of intra-band interactions, the transition from the superconductor to the normal state is continuous as hybridization increases and associated with a quantum critical point. The case when both interactions are present is also considered.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

A new superfluid ground state originally named interior gap or breached pairing (BP) superfluidity has recently been investigated [1–3]. This state presents a homogeneous mixture of normal and superfluid properties and should occur in fermionic systems with different Fermi surfaces. Superfluidity develops at the Fermi surface of the quasi-particles with the smallest Fermi momentum. Since its proposal, much work has been done in understanding the nature of this state and, in particular, its stability [2, 4]. In this work, we consider the possibility of interior gap superfluidity in systems where the quasi-particles can transmute into one another and only their total number is conserved. Our results are directly relevant for condensed matter systems, cold-atom systems [5] in the presence of Rabi coupling [3], and should be of interest for the study of color superconductivity on the core of neutron stars with quarks that can interchange their flavors [4, 6–8]. In cold 2-flavor quark matter, besides *uu* and *dd* pairings, there is also *ud* pairing, since weak processes can supply a hybridization between *u* and *d* quarks.

For concreteness, we focus in the former problem, specifically on superconductivity in transition metals (TM) or rare-earth inter-metallic systems where a large a-band of conduction electrons (s, or p) coexists with a narrow b-band of d- or f-electrons. We consider inter-and intra-band attractive interactions. In both cases we show that a finite interaction is necessary to give rise to superconductivity, differently from the Bardeen–Cooper–Schrieffer (BCS) [9] case. For inter-band attraction the transition into the superconducting state is first order. We find a new metastable superconducting state with features of the internal gap or breached pairing state [2], including *Fermi surfaces* with gapless excitations. In the intra-band case, there is a superconducting quantum critical point (QCP) that can be probed in experiments under pressure. Finally, we include both inter-and intra-band interactions, and show that in this case gapless excitations are generally suppressed.

2. Inter-band superconductivity

We initially consider a model with two types of quasi-particles, a and b, with an attractive interaction [10] g and a hybridization term V that mixes different quasi-particle states. This one-body mixing term V may be tuned by external parameters, allowing us to explore the phase diagram and quantum phase transitions of the model. The Hamiltonian is given by,

$$H = \sum_{k\sigma} \epsilon_k^a a_{k\sigma}^\dagger a_{k\sigma} + \sum_{k\sigma} \epsilon_k^b b_{k\sigma}^\dagger b_{k\sigma} + g \sum_{kk'\sigma} a_{k'\sigma}^\dagger b_{-k'-\sigma}^\dagger b_{-k-\sigma} a_{k\sigma} + \sum_{k\sigma} V_k (a_{k\sigma}^\dagger b_{k\sigma} + b_{k\sigma}^\dagger a_{k\sigma}) \quad (1)$$

where $a_{k\sigma}^\dagger$ and $b_{-k'-\sigma}^\dagger$ are creation operators for the light a and the heavy b-quasi-particles, respectively. The index $\ell = a, b$. The dispersion relations $\epsilon_k^\ell = k^2/2m_\ell - \mu_\ell$ and the ratio between effective masses is taken as $\alpha = m_a/m_b < 1$.

The V-term is responsible for the transmutation among the quasi-particles. In metallic systems, such as transition metals [11], inter-metallic compounds and heavy fermions [12], it arises from mixing of the wavefunctions of the quasi-particles through the crystalline potential. In the quark problem, it is the weak interaction that allows the transformation between up and down quarks and gives rise to the mixing term [6–8]. For a system of cold fermionic atoms in an optical lattice, with two atomic states (a and b), the V-term is due to Raman transitions with an effective Rabi frequency which is directly proportional to V [3]. Then, the hybridization term is included to take into account these effects that allow for a quasi-particle (a or b) transform into one another, such that only the total number of particles ($a + b$) is conserved. The physical origin of the V-term is different for each of the systems, as described above. In the metallic case, which is our main interest here, hybridization can be controlled easily through an applied pressure that varies the overlap between the atomic wavefunctions. In this way it provides a very useful control parameter that can be changed externally, allowing us to probe experimentally the phase diagram of these materials.

When $V = 0$ this model requires a critical value Δ_{ab}^c of the order parameter, $\Delta_{ab} = -g \sum_k \langle a_k b_{-k} \rangle$, to sustain BCS superconductivity [1] (we neglect spin indexes here). The instability of the BCS phase for $\Delta_{ab} < \Delta_{ab}^c$ is associated with a soft mode at a wavevector k_c ($k_F^a < k_c < k_F^b$) which suggests a transition to a Fulde and Ferrel, Larkin, Ovchinnikov (FFLO) state [13] with a characteristic wavevector $k = k_c$. However, the window of parameters for which this phase is stable is very narrow [14] and a BP or Sarma phase [1, 15] has also been considered. Since this corresponds to a maximum of the free energy, a mixed phase with normal and superconducting regions [4] was proposed as an alternative ground state for $\Delta_{ab} < \Delta_{ab}^c$.

In order to obtain the spectrum of excitations of equation (1) within the BCS (mean-field) approximation, we use the equation of motion method to calculate standard and anomalous Green's functions [16]. Excitonic types of correlations that just renormalize the hybridization [17] have been neglected. For the purpose of obtaining the order parameter Δ_{ab}, it is necessary to calculate the anomalous Green's function, $\langle\langle a_k; b_{-k} \rangle\rangle$. When we write the equation of motion for this Green's function, new Green's functions are generated [16]. Some of these are of higher order, as they contain a larger number of creation and annihilation operators than just the two initial Green's function. For these, we apply a BCS type of decoupling [16] to reduce them to the order of the original propagator. Finally, writing the equations of motion for the new Green's functions, we obtain a closed system of equations that can be solved. The anomalous propagator from which the order parameter is self-consistently obtained is given by,

$$\langle\langle a_k; b_{-k} \rangle\rangle = \frac{-\Delta_{ab} \left[(\omega - \epsilon_k^b)(\omega + \epsilon_k^a) + (V^2 - \Delta_{ab}^2) \right]}{(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)}. \quad (2)$$

Besides, hybridization combined with the interaction g can give rise to a net attraction between the b quasi-particles, even in the absence of such interaction in the original Hamiltonian. This becomes manifest in the calculations where we find a finite anomalous Green's function $\langle\langle b_k; b_{-k} \rangle\rangle$ given by,

$$\langle\langle b_k; b_{-k} \rangle\rangle = \frac{-2\Delta_{ab} V \epsilon_k^a}{(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)}. \quad (3)$$

However, it turns out that the anomalous correlation function $\langle b_k b_{-k} \rangle$ obtained from the above propagator is identically zero.

The poles of the Green's function, equation (2), occur for $\omega = \pm\omega_{12}(k)$, where,

$$\omega_{12}(k) = \sqrt{A_k \pm \sqrt{B_k}} \quad (4)$$

with,

$$A_k = \frac{(\epsilon_k^{a2} + \epsilon_k^{b2})}{2} + (V^2 + \Delta_{ab}^2) \quad (5)$$

and

$$B_k = \frac{(\epsilon_k^{a2} - \epsilon_k^{b2})^2}{4} + (\epsilon_k^a + \epsilon_k^b)^2 V^2 + 4V^2 \Delta_{ab}^2 + (\epsilon_k^a - \epsilon_k^b)^2 \Delta_{ab}^2. \quad (6)$$

In the calculations below, we take $\hbar^2/(2m_a\mu_a) = 1$, since the relevant parameter is the mass ratio α. Energies are normalized by the Fermi energy μ_a of the light quasi-particles, such that in all figures the quantities in the axis are numbers. The original band dispersion relations are then written as $\epsilon_k^a = k^2 - 1$ and $\epsilon_k^b = \alpha k^2 - b$. Assuming that all states with negative energy are filled, we have $k_F^a = 1$. We take $k_F^b = 1.45$, $\alpha = 1/7$, such that $\mu_b/\mu_a = b \approx 0.30$, as in [4] for cold atomic systems[1]. These numbers are also appropriate to describe transition metals (TM) for which typical values of the bandwidths ($\mu_{a,b}$) are a few electron-volts, with g and V both of order 10^{-1} or 10^{-2}. The mass ratio α ranges from 10^{-1} for TM to 10^{-3} for heavy fermions (HF) [18]. The general features of the solutions that we obtain are, however, independent of a particular set of parameters. Figure 1 shows the dispersion relations of the excitations. Differently from the case $V = 0$, there are no negative values of the energy [1] for any $\Delta_{ab} \neq 0$. However, the dispersion relations vanish at two two-dimensional *Fermi surfaces* [19] determined by,

$$\epsilon_k^a \epsilon_k^b + (\Delta_{ab}^2 - V^2) = 0 \quad (7)$$

[1] Throughout this work, V is sufficiently small, such that both quasi-particle states are occupied ($V \lesssim 0.95$).

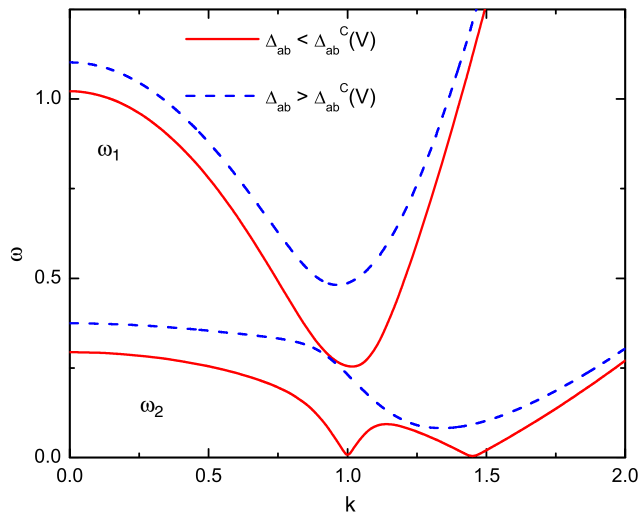

Figure 1. Dispersion relations for $V = 0.1$: $\Delta_{ab} = 0.1 < \Delta^c_{ab}(V = 0.1) \sim 0.224$ (full line) and $\Delta_{ab} = 0.35 > \Delta^c_{ab}(V = 0.1)$ (dashed line).

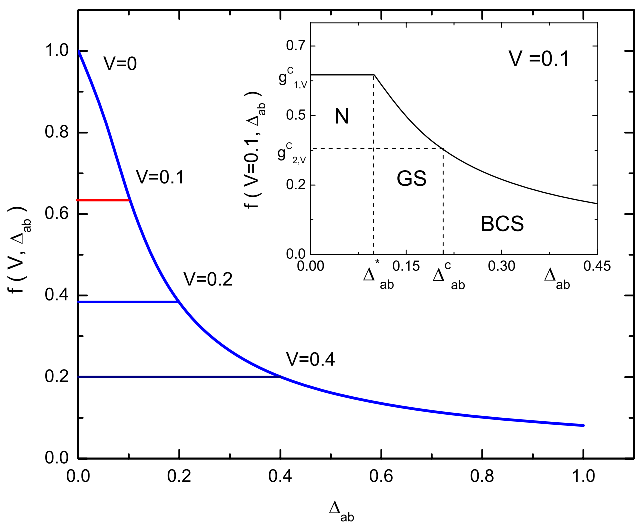

Figure 2. Gap function f normalized to its value at $V = 0$, for different values of hybridization V. The inset shows the phases associated with different values of the order parameter Δ_{ab} for a fixed hybridization $V = 0.1$. N is a normal phase, and GS and BCS correspond to gapless and BCS superconducting phases, respectively. The interactions $g^c_{1,V}$ and $g^c_{2,V}$ mark the limits of the gapless (GS) and BCS superconducting phases (see figure 3).

for $\Delta_{ab} \leqslant \Delta^c_{ab}(V)$ where,

$$\Delta^c_{ab}(V) = \sqrt{\Delta^c_{ab}(V = 0)^2 + V^2} \tag{8}$$

with [4] $\Delta^c_{ab}(V = 0) = |(\alpha - b)|/2\sqrt{\alpha}$. As Δ_{ab}, i.e. the coupling g increases and reaches $\Delta^c_{ab}(V)$, the two gapless Fermi surfaces (FS) merge at a critical FS. For $\Delta_{ab} > \Delta^c_{ab}(V)$ the dispersion relations are BCS-like with a finite gap for excitations (see figure 1). The instability of the BCS phase can also be triggered by the hybridization, which increases the mismatch of the Fermi surfaces due to a *repulsion* between the bands [6]. It occurs at a critical value, $V_c = \sqrt{\Delta^2_{ab} - \alpha(k_F^{b2} - k_F^{a2})^2/4}$ for a fixed $\Delta^2_{ab} > \alpha(k_F^{b2} - k_F^{a2})^2/4$. Both instabilities, due to increasing hybridization or by decreasing the coupling g (or Δ_{ab}), belong to the same universality class and are associated with a soft mode at a wavevector k_c.

Dispersion relations with similar features of those shown in figure 1 were obtained for color superconductivity [19]. An additional p-wave instability at the new FS [20], which is outside the scope of the present mean-field approach, has been investigated. In the metallic problem there is the possibility of additional pairing in the s-wave channel of the same type of particles due to the extra spin degree of freedom (see equation (3)). However, as pointed out before, the relevant anomalous correlation function associated with this Green's function turns out to be identically zero. Notice that the dispersion of the fermions close to the new FS are linear and, at least in $d = 2$, this requires a finite interaction for pairing to occur [21]. It would be interesting to consider other types of instability at these Fermi surfaces, such as spin density wave ordering.

From the discontinuity of the Green's functions on the real axis we can obtain the anomalous correlation function characterizing the superconducting state. The self-consistent equation for the order parameter $\Delta_{ab} = -g\sum_k \langle b_{-k}a_k\rangle$ at $T \neq 0$ is given by,

$$\frac{1}{g} = \sum_{j=1}^{2} \int \frac{d^3k}{(2\pi)^3}\left[\frac{(-1)^j}{2\sqrt{B_k}}\left(\frac{\omega_j(k)^2 - E^2(k)}{2\omega_j(k)}\right) \times \tanh\left(\frac{\beta\omega_j(k)}{2}\right)\right] \tag{9}$$

where $E^2(k) = \epsilon^a_k\epsilon^b_k + (\Delta^2_{ab} - V^2)$. This equation can be written as $1/g\rho = f(V, \Delta_{ab})$, where ρ is the density of states at the Fermi level of the a-band. The function $f(V, \Delta_{ab})$ is plotted in figure 2 for several values of the hybridization parameter. For $V = 0$ a solution with a finite order parameter Δ_{ab} only exists for $(1/g) < (1/g^c_1) = \rho f(0, 0)$ with $f(0, 0) = (2/(1-\alpha))|\ln[(b-\alpha)/(\omega_c(1-\alpha)+(b-\alpha))]| \sim 0.123$. The quantity $\omega_c = 0.01$ is a small cut-off energy around the Fermi energy where the integrals in energy are performed. Still, for $V = 0$, there is another characteristic value of the coupling $(1/g^c_2) = \rho f(0, \Delta^c_{ab}(V = 0))$, such that for $g^c_1 < g < g^c_2$ the system presents a BP or a mixed phase [4]. For $g > g^c_2$, superconductivity is of the BCS type [4]. Since the BP phase appears as a maximum of the free energy, an alternative state for $g^c_1 < g < g^c_2$ is a mixed phase with coexisting normal and superconducting BCS-like regions [4]. For $g > g^c_2$ the superconducting BCS is the stable ground state [4].

As hybridization is turned on at zero temperature, a stronger value of the coupling g is necessary to obtain a superconducting solution, since $f(V, 0) < f(0, 0)$ (figure 2). The function $f(V, \Delta_{ab})$ normalized by its value for $V = 0$ is shown in figure 2. Although hybridization acts to the detriment of superconductivity, we notice that, at least for small values of V, a weak coupling approximation is still justified, as for $V = 0$ treated in [1]. The function $f(V, \Delta_{ab})$

is flat up to $\Delta_{ab} = \Delta^{*}_{ab}(V) \sim V$ (see figure 2), such that, when the coupling g is strong enough to stabilize a superconducting solution, it occurs already at a finite value of the order parameter. Consequently, for $V \neq 0$ the quantum normal to the superconducting phase transition as a function of the coupling g is first order. For $\Delta^{*}_{ab}(V) < \Delta_{ab} < \Delta^{c}_{ab}(V)$ there is a superconducting solution, the GS phase in figure 2, with the spectra of excitations shown in figure 1 as full lines. This solution corresponds to a metastable minimum of the free energy. This is shown in figure 3, where we plot the zero-temperature free energy for a fixed hybridization, $V = 0.1$, and different values of the coupling parameter g. The metastable minima appear for $g^{c}_{2,V} > g > g^{c}_{1,V}$ and occur at values of the order parameter of $\Delta^{*}_{ab}(V) < \Delta_{ab} < \Delta^{c}_{ab}(V)$, as shown in figure 3. For these values of Δ_{ab} the gaps in the lower branch of the dispersion relations vanish at two two-dimensional *Fermi surfaces* (see figure 1). This superconducting phase has similarities to the BP superconductor [1] in that both have gapless excitations, but with the difference that the present one corresponds to a minimum, even though metastable, of the free energy. At $g = g^{c}_{2,V}$ the normal and superconducting phase exchange stability at a quantum first-order phase transition. The critical value $g^{c}_{2,V}$ for a fixed V is given by the condition $E[\Delta_{ab}(V) = 0, g = g^{c}_{2,V}] = E[\Delta_{ab}(V), g = g^{c}_{2,V}]$, where $E(\Delta_{ab}, V, g)$ is the zero-temperature free energy. As the coupling g increases beyond $g^{c}_{1,V}$, the first solution for this equation is obtained for $\Delta_{ab}(V) = \Delta^{c}_{ab}(V)$ (see equation (8) and figure 3). Thus, the first-order transition *as a function of the coupling strength* occurs together with the change in the excitation spectrum. The ground-state free energy has a kink at the critical value $g = g^{c}_{2,V}$ at which the quantum first-order transition occurs [22]. For $g > g^{c}_{2,V}$ the stable ground state is a BCS type of superconductor with gapped excitations, since the stable free energy minimum occurs for values of the order parameter of $\Delta_{ab} > \Delta^{c}_{ab}(V)$ (see figure 3). The dispersion relations are like those shown as dashed lines in figure 1. We point out that for $g \leqslant g^{c}_{1,V}$ ($\Delta \leqslant \Delta^{*}_{ab}(V)$) the metastable minimum of the free energy disappears (figure 3). Then, the value $g = g^{c}_{1,V}$ marks the limit of stability of the BCS-like superconducting phase into the normal phase. The other limit, of the metastable normal phase in the superconducting phase, is not shown. Then, as one increases hybridization in a two-band BCS superconductor with attractive inter-band interactions, two main effects take place. First, hybridization increases the mismatch between the Fermi surfaces, giving rise to a first-order transition from the BCS superconductor to the normal state. At this transition there appears a metastable GS phase with two two-dimensional Fermi surfaces with gapless excitations. Differently from the breached pairing state, in this GS phase pairing takes place among quasi-particles with momenta between k^{a}_{F} and k^{b}_{F}. The mixing of the quasi-particles allows them to take advantage of the condensation energy in this range of k-space, reducing the energy of the GS phase with respect to the BP state.

3. Intra-band interactions

Next we consider a closely related model which is relevant for many physical systems of interest, such as inter-metallic compounds, high-T_{c} and heavy-fermion materials [23]. It consists of a narrow band of quasi-particles with an attractive interaction that hybridizes with another band. The Hamiltonian is given by,

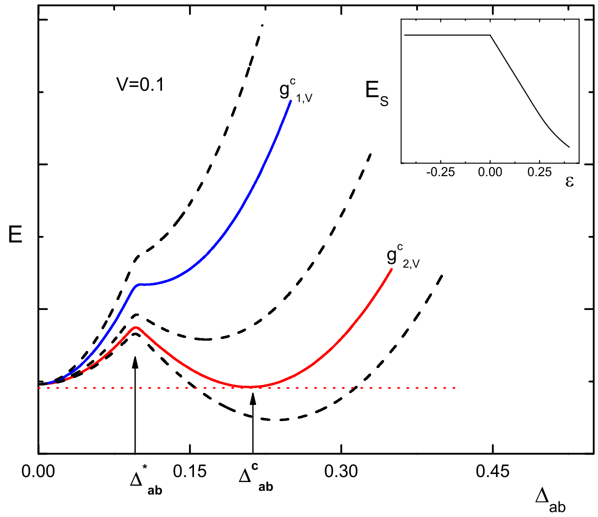

Figure 3. Free energy at zero temperature as a function of the order parameter for different values of the interaction g and a fixed hybridization $V = 0.1$. For $g^{c}_{2,V} > g > g^{c}_{1,V}$ there is a metastable superconducting (GS) phase with $\Delta^{c}_{ab}(V) > \Delta_{ab} > \Delta^{*}_{ab}(V)$ and gapless excitations. Inset shows the ground-state energy as a function of $\epsilon = g - g^{c}_{2,V}$. The ground-state energy has a kink [22] at the quantum first-order transition at the critical value $g = g^{c}_{2,V}$, or at $\Delta_{ab} = \Delta^{c}_{ab}$ if plotted as a function of the order parameter.

$$H = \sum_{k\sigma} \epsilon^{a}_{k} a^{\dagger}_{k\sigma} a_{k\sigma} + \sum_{k\sigma} \epsilon^{b}_{k} b^{\dagger}_{k\sigma} b_{k\sigma} + g_{\mathrm{b}} \sum_{kk'\ \sigma} b^{\dagger}_{k'\sigma} b^{\dagger}_{-k'-\sigma} b_{-k-\sigma} b_{k\sigma} + \sum_{k\sigma} V_{k} (a^{\dagger}_{k\sigma} b_{k\sigma} + b^{\dagger}_{k\sigma} a_{k\sigma}). \tag{10}$$

In this case we have to keep track of the spin indexes, since the operators associated with the particles forming the pairs do not necessarily anticommute. The dispersion relations of the quasi-particles in the BCS approximation are obtained, as before, from the poles of the Green's functions. They are given by $\omega_{12}(k) = \sqrt{\tilde{A}_{k} \pm \sqrt{\tilde{B}_{k}}}$ with,

$$\tilde{A}_{k} = \frac{\epsilon^{a2}_{k} + \epsilon^{b2}_{k}}{2} + V^{2} + \frac{\Delta^{2}}{2} \tag{11}$$

and

$$\tilde{B}_{k} = \left(\frac{\epsilon^{b2}_{k} - \epsilon^{a2}_{k} + \Delta^{2}}{2} \right)^{2} + V^{2} \left[(\epsilon^{a}_{k} + \epsilon^{b}_{k})^{2} + \Delta^{2} \right] \tag{12}$$

where $\Delta = -g_{\mathrm{b}} \sum_{k} \langle b_{-k\uparrow} b_{k\downarrow} \rangle$ is a new order parameter associated with superconductivity in the narrow b-band. For $V \neq 0$, the dispersion relations above do not vanish for any value of k, as can be verified from the condition,

$$Z(k) = \tilde{A}^{2}_{k} - \tilde{B}_{k} = (\epsilon^{a}_{k} \epsilon^{b}_{k} - V^{2})^{2} + \Delta^{2} \epsilon^{a2}_{k} = 0 \tag{13}$$

which has no real solution. These new dispersions are shown in figure 4. The lower branch of the dispersion has dips for

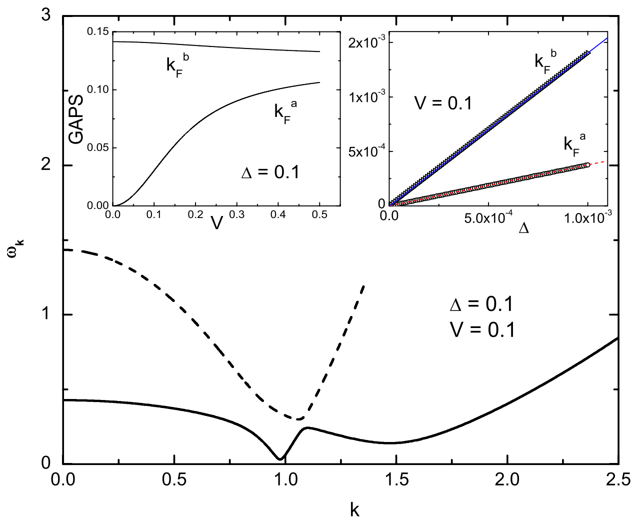

Figure 4. Dispersion relations for model equation (10). Inset shows the energy of the minima in the lower dispersion close to k_F^a and k_F^b as a function of Δ and V.

wavevectors close to the original Fermi wavevectors. The gaps at the dips vary linearly with the order parameter Δ, for fixed V, as shown in the inset. This suggests that the modes at the dips behave as roton-like excitations, with a roton gap proportional to the superconducting order parameter. For fixed Δ changing the hybridization, the gap close to k_F^a can become arbitrarily small (inset of figure 4). As shown in this figure, this gap may be smaller than the gap at k_F^b associated with superconductivity. This has experimental consequences, as the activated behavior of thermodynamic properties will be dominated by the smaller gap due to hybridization. The gap equation at $T = 0$ is given by,

$$\frac{1}{g_b\rho_b} = f_b(\Delta, V) = \frac{1}{2}\int_{-\omega_0}^{\omega_0} d\epsilon \frac{1}{\omega_1(\epsilon)+\omega_2(\epsilon)} \times \left[1 + \frac{(\epsilon + (b-\alpha))^2}{\alpha^2\sqrt{Z(\epsilon)}}\right] \tag{14}$$

where ρ_b is the density of states of the narrow b-band at the Fermi level. For $V = 0$ this reduces to the BCS gap equation for a single b-band. In figure 5 we show $f_b(V, \Delta)$ as a function of Δ for several values of the hybridization. We find that $f_b(V, 0)$ is finite for values of $V \neq 0$, showing that in this case a finite interaction $g_b^c(V) = 1/(\rho_b f_b(V, 0))$ is necessary for the appearance of superconductivity differently from a single (usual) BCS band. Notice that for physical values of the hybridization, $V \leqslant 0.12$, the condition for superconductivity $g_b^c(V)\rho_b < 1$ is still in the weak coupling regime (see figure 5). Then, for small but reasonable values of V, the present BCS approach yields useful results. As in the previous section, in this intra-band case we get a finite Green's function $\langle\langle a_{k\uparrow}; b_{-k\downarrow}\rangle\rangle$, but we find that the anomalous correlation function $\langle b_{-k\downarrow} a_{k\uparrow}\rangle$ is identically zero.

The quantum phase transition at $g_b^c(V)$ is second order, as can be seen from figure 5, since the condition $1/g_b^c(V)\rho_b = f_b(V, \Delta))$ is first satisfied for $\Delta = 0$. Besides, the free energy curves in the inset of this figure show directly the continuous nature of the transition. Quantum fluctuations such

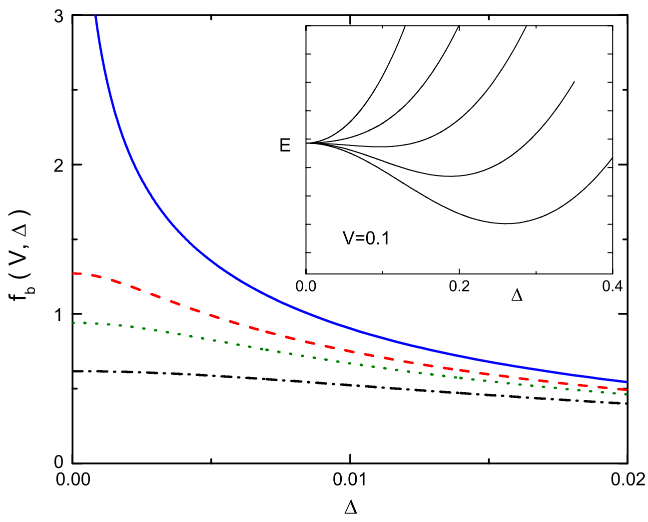

Figure 5. Gap function $f_b(V, \Delta)$ for different values of hybridization ($V = 0.10, 0.12, 0.13$ and 0.15 from top to bottom). Inset: free energy ($T = 0$) as a function of the order parameter for different values of the coupling g_b. As this increases, the minimum moves continuously from $\Delta = 0$ to a finite value as the system enters the superconducting phase. Similar curves are obtained, but with the minimum moving to $\Delta = 0$, if V is increased, starting from V_0 for a fixed $g_b > g_b^c(V_0)$.

as coupling to the electromagnetic field [22] could eventually drive this transition first order, but this is outside the scope of the present BCS approximation. Since in real multi-band systems some hybridization always occurs, the existence of a quantum critical point should be ubiquitous in superconducting compounds with intra-band attractive interactions. This QCP can be reached by applying pressure in the system to vary the overlap of the atomic orbitals and consequently V, as is common, for example, in the study of HF materials [18].

4. Intra- and inter-band case

Finally, we address the general case of attraction among the heavy b-quasi-particles and the a and b fermions (inter- and intra-band attractive interactions [8]) in the presence of hybridization. The calculations are long but can be carried out analytically. The new excitations are obtained from the equation,

$$\omega^4 - \left[\epsilon_k^{a2} + \epsilon_k^{b2} + 2(V^2 + \Delta_{ab}^2) + \Delta^2\right]\omega^2 + 4V\Delta\Delta_{ab}\omega + \left[\epsilon_k^a\epsilon_k^b - (V^2 - \Delta_{ab}^2)\right]^2 + \Delta^2\epsilon_k^{a2} = 0. \tag{15}$$

For the frequency of these excitations to vanish it is required that $[\epsilon_k^a\epsilon_k^b - (V^2 - \Delta_{ab}^2)]^2 + \Delta^2\epsilon_k^{a2} = 0$. This can occur by tuning the hybridization parameter, such that $V = \Delta_{ab}$, in which case gapless excitations appear at $k = k_F^a$, where $\epsilon_{k=k_F^a}^a = 0$. Without this fine tuning there are no gapless modes. If, for symmetry reasons, we neglect the term linear in ω, we obtain the energy of the excitations in the form $\omega_{12}(k) = \sqrt{\bar{A}_k \pm \sqrt{\bar{B}_k}}$ with,

$$\bar{A}_k = A_k + \frac{\Delta^2}{2} \tag{16}$$

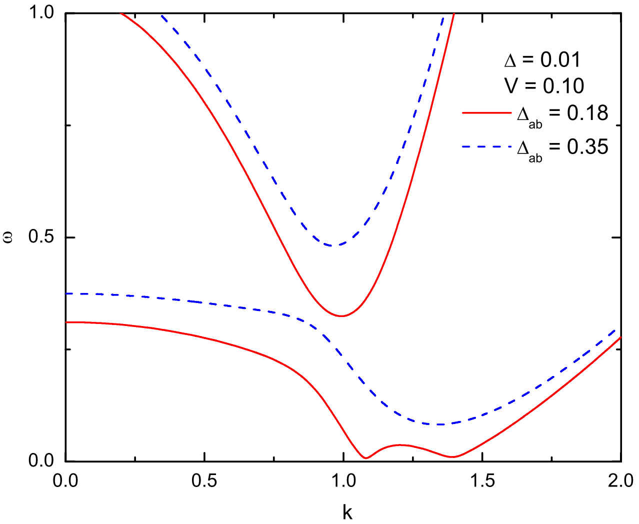

Figure 6. Dispersion relations for the general case (intra-and inter-band attraction). We consider two cases of Δ_{ab} larger and smaller than $\Delta_{ab}^{c}(g_b = 0) \approx 0.2$. In the latter case, the dispersion relation can become very small for wavevectors close to the original Fermi surfaces.

and

$$\bar{B}_k = B_k + \frac{\Delta^4}{4} - \frac{\Delta^2}{2}(\epsilon_k^{a2} - \epsilon_k^{b2}) + \Delta^2(V^2 + \Delta_{ab}^2) \qquad (17)$$

where A_k and B_k are given by equations (5) and (6), respectively. In the appropriate limits these equations reduce to the cases that we studied before. Notice that in this case there are two order parameters in the problem, Δ and Δ_{ab}, both defined before. The dispersion relations are shown in figure 6. Excluding the fine-tuned case $V = \Delta_{ab}$, any attractive interaction among the b-quasi-particles removes the gapless modes in the dispersion relations independently of Δ_{ab} or the Fermi surface mismatch. The order parameters are determined by two coupled equations which for finite temperature are given by,

$$\frac{1}{g\rho} = \frac{-1}{2}\int_{-\omega_0}^{\omega_0}\frac{d\epsilon}{\sqrt{B(\epsilon)}}\left[\left(\frac{\omega_1^2(\epsilon) - \gamma^2(\epsilon)}{2\omega_1(\epsilon)}\right)\tanh\frac{\beta\omega_1(\epsilon)}{2} - \left(\frac{\omega_2^2(\epsilon) - \gamma^2(\epsilon)}{2\omega_2(\epsilon)}\right)\tanh\frac{\beta\omega_2(\epsilon)}{2}\right] \qquad (18)$$

and

$$\frac{1}{g_b\rho_b} = \frac{1}{2}\int_{-\omega_0}^{\omega_0}\frac{d\epsilon}{\sqrt{B(\epsilon)}}\left[\left(\frac{\alpha^2\omega_1^2(\epsilon) - (\epsilon + b - \alpha)^2}{2\alpha^2\omega_1(\epsilon)}\right) \times \tanh\frac{\beta\omega_1(\epsilon)}{2} - \left(\frac{\alpha^2\omega_2^2(\epsilon) - (\epsilon + b - \alpha)^2}{2\alpha^2\omega_2(\epsilon)}\right) \times \tanh\frac{\beta\omega_2(\epsilon)}{2}\right] \qquad (19)$$

where

$$\gamma^2 = \left(\frac{\epsilon + (\alpha\epsilon - b)}{2}\right)^2 + (\Delta_{ab}^2 - V^2) + \frac{\Delta V}{4} \times \left(\Delta V + 4\left(\frac{\epsilon + (\alpha\epsilon - b)}{2}\right)\right) - \left(\frac{\epsilon - (\alpha\epsilon - b)}{2} - \frac{\Delta V}{2}\right)^2. \qquad (20)$$

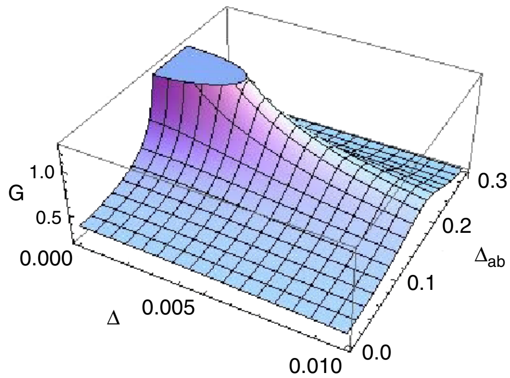

Figure 7. The gap function $\bar{G}(\Delta, \Delta_{ab})$ for $V = 0.15$. For small Δ there is a region of first-order transitions for $\Delta_{ab} \sim V$.

The right-hand sides of equations (18) and (19) define the gap functions $\bar{f}(\Delta, \Delta_{ab})$ and $\bar{f}_b(\Delta, \Delta_{ab})$, respectively. Adding these equations, we get $(1/\rho g) + (1/\rho_b g_b) = \bar{G}(\Delta, \Delta_{ab}) = \bar{f}(\Delta, \Delta_{ab}) + \bar{f}_b(\Delta, \Delta_{ab})$. This function is plotted in figure 7. For $\Delta_{ab} \sim V$ and small values of Δ there is a region of first-order transitions, and this remains valid even as $V \to 0$. The existence of an intra-band interaction and two order parameters makes this case qualitatively different from the pure inter-band interaction, even in the limit $V \to 0$ [10].

5. Conclusions

We have investigated superconductivity in two-band systems with mismatched Fermi surfaces in the presence of hybridization using a mean-field approximation. For inter-band interactions we found a phase with gapless excitations on two two-dimensional *Fermi surfaces*. This replaces the BP phase in the case where the quasi-particles can transmute into one another. This phase corresponds to a metastable minimum of the free energy for a constant q-independent interaction. Differently from the BP phase, pairing occurs between the mismatched Fermi surfaces, and this results in a net gain of energy due to the condensation of these quasi-particles. In the intra-band case we have shown the existence of a QCP at which superconductivity is destroyed as hybridization (pressure) increases beyond a critical value. The phase diagram and quantum phase transitions can be explored by changing either the strength of the attractive interactions or the hybridization. Hybridization, among other things, varies the mismatch of the Fermi surfaces. Since in real systems it can be controlled by external pressure, it is a useful parameter to investigate the effects of Fermi surface mismatch in multi-band superconductors. Our mean-field approach is qualitatively appropriate for treating weak coupling systems with $g, g_b \sim 1$, although even in this case it can miss effects due to fluctuations, such as an additional p-wave instability [24]. In the metallic problem, the quasi-particles have spins as extra degrees of freedom and, in principle, there is the possibility of an additional s-wave pairing between quasi-particles at the gapless Fermi surfaces. This is taken into account in the mean-field approach, even if the interaction between these quasi-particles is not included in the Hamiltonian. This is manifested through the appearance of anomalous Green's functions involving these quasi-particles. However, only in the case where g and g_b are finite do we find two order parameters, with none being identically zero.

In heavy-fermion materials [18, 25] hybridization plays an important role, and they could display the effects and phase transitions discussed above. As hybridization (pressure) increases, giving rise to Fermi surface mismatch, we expect a QCP associated with vanishing superconductivity for predominant intra-band interactions. If inter-band coupling is stronger, an FFLO or some other exotic superconducting phase is expected with increasing hybridization. The origin of the attractive interaction, whether it is due to phonons or spin-fluctuations, does not affect the present results, although the use of a mean-field approximation appears questionable for treating these strongly correlated materials. However, as pointed out in [1], for fixed $k_F^{a,b}$ and inter-band interactions, the critical coupling $g_{1,2}^c \to 0$ as the mass ratio $\alpha \to 0$. Since this holds in the presence of hybridization, HF materials that are characterized by small mass ratios, α, fall in the weak coupling regime, for which the present mean field is appropriate.

Multi-band superconductors such as MgB_2 are also candidates for investigating the effects discussed here [26]. Pressure decreases the temperature of the superconducting transition, although in the present stage of experiments it is not enough to drive them to a QCP. Evidence of topological electronic transitions has been found in these experiments. These transitions involve changes in Fermi surfaces and bear some resemblance [3] to those studied here. We hope that the results presented in this paper will stimulate further experimental work in multi-band superconductors.

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